AP Calculus Summer Review Packet 2019-2020

Name:_____

Dear Future AP Calculus Student,

I hope you are excited for the year of Calculus that we will be pursuing together! We will be having a lot of fun—and doing a lot of work—learning everything there is to know about the mathematics of motion. Calculus can be described as the "mathematics of change"—how fast things change, how to predict change, and how to use information about change to understand different systems themselves. In many ways, Calculus will simply extend what you already know about math one step further.

In order to give you a head start in the understanding of Calculus, I've designed a few things for you to do. The purpose of this assignment is to have you practice the mathematic skills necessary to be successful in Calculus AB. All of the skills covered in this packet are skills learned in Algebra II and Pre-Calculus. If you need to, you may use reference materials to assist you and refresh your memory (old notes, textbooks, online resources, etc.) but keep in mind you must have this material mastered. Most of this packet should be completed without the use of a calculator. I hope that these problems help you to maintain/improve your skills so we can hit the ground running from day one.

AP Calculus AB is a fast paced course that is taught at the college level. There is a lot of material in the curriculum that must be covered before the AP exam in May. Therefore, we cannot spend a lot of class time reteaching prerequisite skills. This is why you have this packet. Try your best to complete this packet on your own. It would be in your best interest to not copy others or try and cheat. Again, all of the material covered in this packet needs to be known in order to be successful in our class next year. Try to pace the assignment over the entirety of the summer to maintain the requisite skills needed to do well in the course.

This packet will be due very soon after school starts. You will have some time to ask me questions when school starts but we will begin new material on the first day. That being said, you should attempt to get the majority of the packet done before school starts otherwise your first week might prove very difficult.

This year you will be expected recall the unit circle and use it from memory this is why I have attached a unit circle as the second page to this packet. I will post a video explaining a good way to memorize the unit circle that has helped me over the years. This video will be posted on my google classroom account. The account password is 6s64u7.

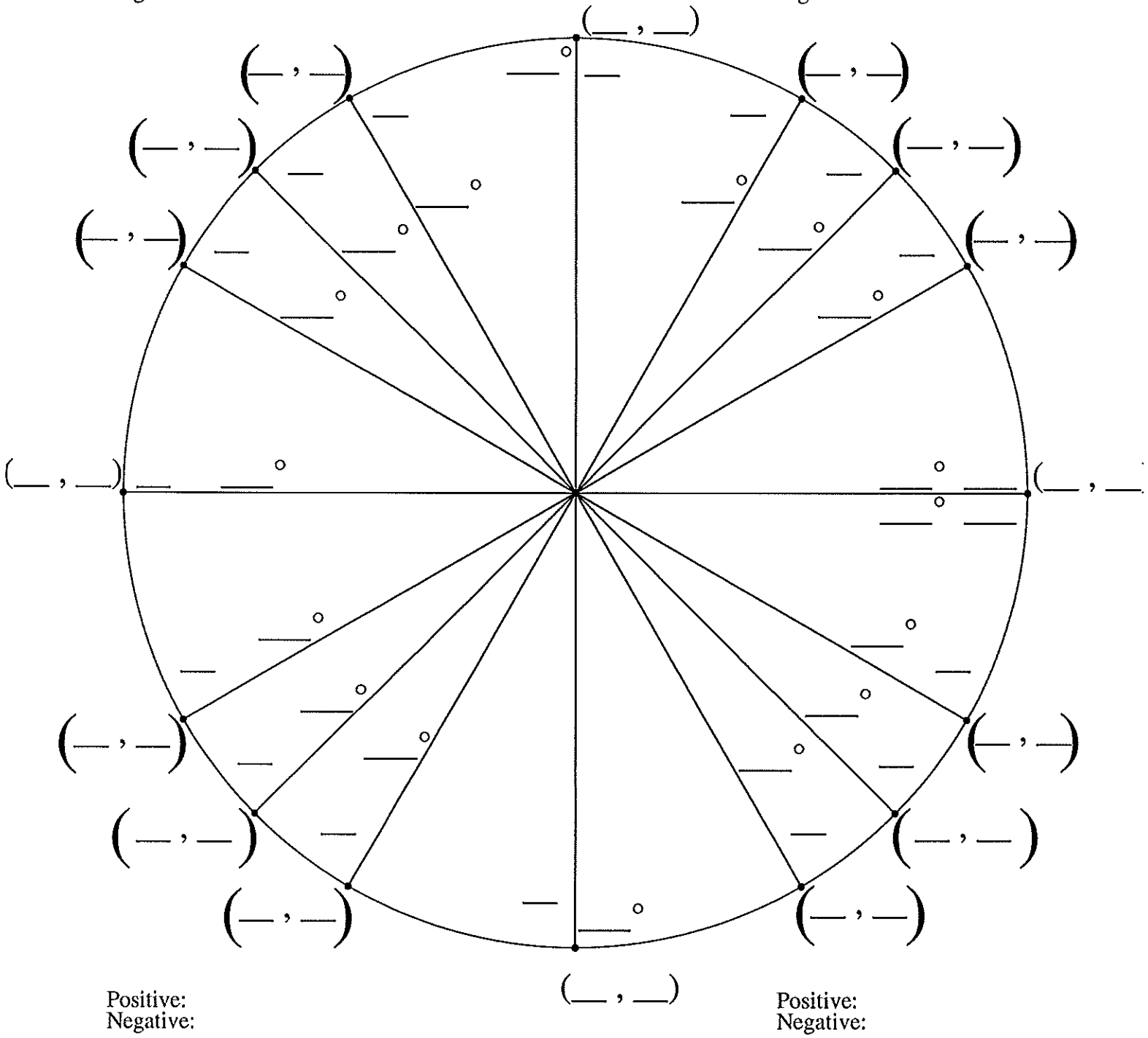
Questions 101-116 you likely have not seen before. One video you may find helpful is located at https://www.youtube.com/watch?v=UkjgJQaGx98. There are a plethora of other videos that may be of use on khan academy or youtube if this video does not help you sufficiently.

Be sure to show your work on all of these problems. If you have any questions about the problems feel free to email my anytime over the summer at daniel.kreilein@evsck12.com.

I hope you all have a great summer!

Mr. Daniel Kreilein

Fill in The Unit Circle



EmbeddedMath.com

Useful Formulas

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

$\cos 2x = \cos^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

Logarithms:
$y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \frac{\log_b n}{\log_b a}$

Fractional exponent: $\sqrt[b]{x^c} = x^{\frac{c}{b}}$

Negative Exponents: $x^{-n} = 1/x^n$

The Zero Exponent: $x^0 = 1$, for x not equal to 0.

Multiplying Powers

Multiplying Two Powers of the Same Base:
$(x^a)(x^b) = x^{(a+b)}$

Multiplying Powers of Different Bases:
$(xy)^a = (x^a)(y^a)$

Dividing Powers

Dividing Two Powers of the Same Base:
$(x^a)/(x^b) = x^{(a-b)}$

Dividing Powers of Different Bases:
$(x/y)^a = (x^a)/(y^a)$

Slope-intercept form: $y = mx + b$

Point-slope form: $y = m(x - x_1) + y_1$

Standard form: $Ax + By + C = 0$

Keep in mind, all of these formulas are ones you will NEED TO KNOW for AP Calculus.

Name: _____

Simplify the following:

1. $\frac{3}{4x^2-25} + \frac{2}{2x+5}$

2. $\frac{8}{y-2} - \frac{3y}{2y^2-8}$

3. $\frac{30x^2+53x+22}{70x^2+17x-66}$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

4. $f(2)$

5. $g(-3)$

6. $f(h + 1)$

7. $f[g(-2)]$

8. $g[f(m + 2)]$

9. $[f(x)]^2 - 2g(x)$

Let $f(x) = \sin(2x)$. **Find each EXACT value.**

10. $f\left(\frac{\pi}{4}\right)$

11. $f\left(\frac{2\pi}{3}\right)$

12. $f\left(-\frac{\pi}{6}\right)$

Let $f(x) = x^2,\ g(x) = 2x + 5,\ and\ h(x) = x^2 - 1$. **Find each.**

13. $h[f(-2)]$

14. $f[g(x-1)]$

15. $g[h(x^3)]$

Find the x and y intercepts for each of the following functions:

16. $y = 2x - 5$

17. $y = x^2 + x - 2$

18. $y = x\sqrt{16 - x^2}$

19. $y^2 = x^3 - 4x$

Find the point(s) of intersection of the graphs for the given equations.

20. $\begin{matrix} x+y=8 \\ 4x-y=7 \end{matrix}$

21. $\begin{matrix} x^2+y=6 \\ x+y=4 \end{matrix}$

22. $\begin{matrix} x=3-y^2 \\ y=x-1 \end{matrix}$

Find the domain and range of the following functions. Write your answers in INTERVAL NOTATION.

23. $f(x) = x^2 - 5$

24. $f(x) = -\sqrt{x+3}$

25. $f(x) = 3\sin x$

26. $f(x) = \frac{2}{x-1}$

Find the inverse for each function.

27. $f(x) = 2x + 1$

28. $f(x) = \frac{x^2}{3}$

29. $g(x) = \frac{5}{x-2}$

30. $y = \sqrt{4 - x} + 1$

31. If the graph of $f(x)$ has the point $(2, 7)$ then what is one point that will be on the graph of $f^{-1}(x)$?

32. Explain, in words, how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

33. Determine the equation of a line passing through the point $(5, -3)$ with an undefined slope.

34. Determine the equation of a line passing through the point $(4, -2)$ with a slope of 0.

35. Use point-slope form to find the equation of the line passing through the point $(0, 5)$ with a slope of 2/3.

36. Use point-slope form to find a line passing through the point $(2, 8)$ and parallel to the line $y = \frac{5}{6}x - 1$

37. Use point-slope form to find a line perpendicular to $y = -2x + 9$ passing through the point (4, 7).

38. Find the equation of a line passing through the points (-3, 6) and (1, 2).

39. Find the equation of a line with an x-intercept of (2, 0) and a y-intercept of (0, 3).

Determine the exact value of the following. DO NOT use a calculator.

40. $\sin \pi$

41. $\cos \frac{3\pi}{2}$

42. $\sin\left(-\frac{\pi}{2}\right)$

43. $\sin\left(\frac{5\pi}{4}\right)$

44. $\cos \frac{\pi}{4}$

45. $\cos(-\pi)$

46. $\cos\left(\frac{\pi}{3}\right)$

47. $\sin \frac{5\pi}{6}$

48. $\cos \frac{2\pi}{3}$

49. $\tan\left(\frac{\pi}{4}\right)$

50. $\tan \pi$

51. $\tan\left(\frac{\pi}{3}\right)$

52. $\cos \frac{4\pi}{3}$

53. $\sin \frac{11\pi}{6}$

54. $\tan \frac{7\pi}{4}$

55. $\sin\left(-\frac{\pi}{6}\right)$

Solve each of the equations for $0 \leq x \leq 2\pi$.

56. $\sin x = -\frac{1}{2}$

57. $2\cos x = \sqrt{3}$

58. $4\,sin^2x = 3$

*Recall $sin^2x = (\sin x)^2$

59. $2\,sin^2x - 3\,sinx - 2 = 0$

60. $cos^2x = 1 - sinx$

61. $\sin x - 2\sin x \cos x = 0$

62. Given $f(x) = x^2$ *and* $g(x) = (x-3)^2 + 1$, how does the graph of $g(x)$ differ from $f(x)$ in terms of transformations of functions?

63. Write an equation for the function that has the shape of $f(x) = x^3$ but moved six units to the left and reflected over the x-axis.

Find the vertical asymptotes for the following functions:

64. $f(x) = \frac{1}{x^2}$

65. $f(x) = \frac{x^2}{x^2-4}$

66. $f(x) = \frac{2+x}{x^2(1-x)}$

67. $f(x) = \frac{4-x}{x^2-16}$

68. $f(x) = \frac{x-1}{x^2+x-2}$

69. $f(x) = \frac{5x+20}{x^2-16}$

Find the horizontal asymptotes for the following functions:

70. $f(x) = \frac{x^2-2x+1}{x^3+x-7}$

71. $f(x) = \frac{5x^3-2x+8}{4x-3x^3+5}$

72. $\frac{4x^2}{3x^2-7}$

73. $f(x) = \frac{(2x-5)^2}{x^2-x}$

74. $f(x) = \frac{-3x+1}{\sqrt{x^2+x}}$

***Remember $\sqrt{x^2} = \pm x$

Solve each logarithmic or exponential equation below. Give exact answers.

87. $\log_{20}(8-2x) = \log_{20}(-3x+10)$

88. $5\ln(b-9) = 20$

89. $3\ \log_4(4n-5)+4 = 7$

90. $\log(x+21) + \log x = 2$

91. $5^{2x} = 20$

92. $4 - 2e^{x+1} = -12$

State whether the following functions are even, odd, or neither. Show your work to explain your answer.

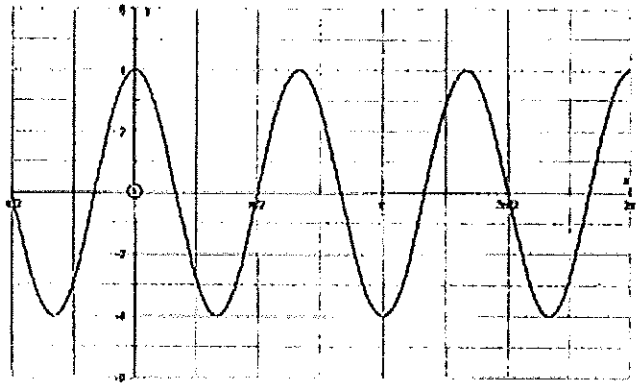
93. $y = 2x^4 - 5x^2$

94. $g(x) = x^5 - 3x^3 + x$

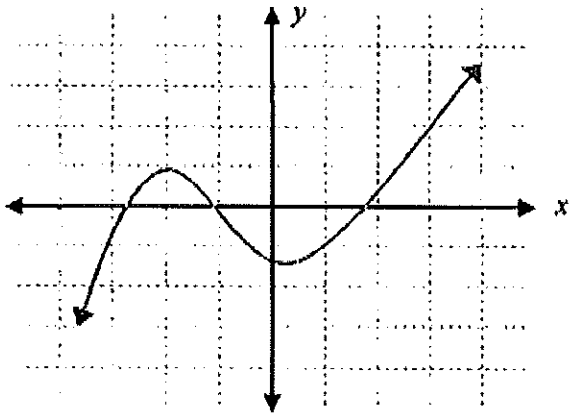
95. $h(x) = 2x^2 - 5x + 3$

96. $f(x) = 2\cos x$

97.

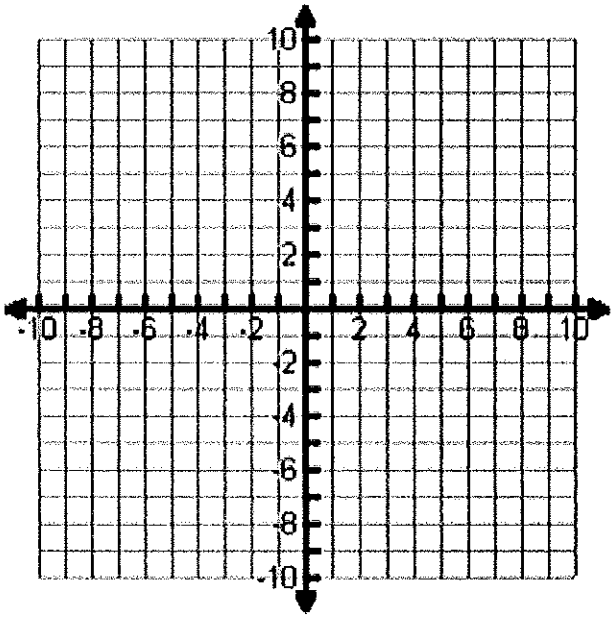


98.

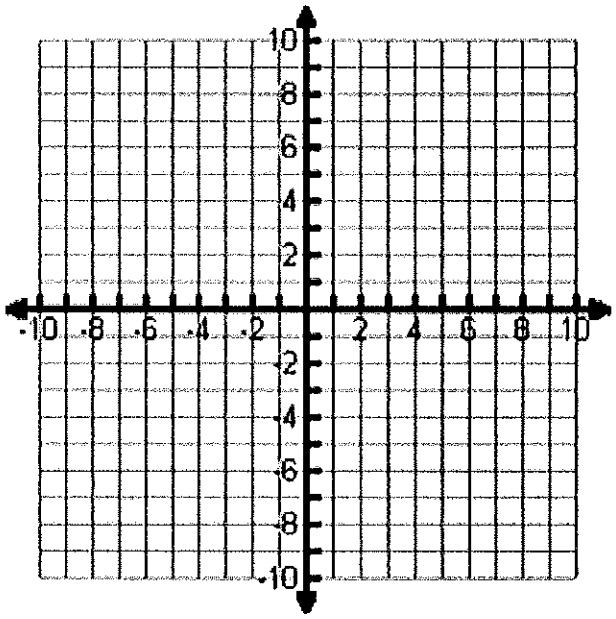


Graph each of the following piecewise functions. Determine any points of discontinuity.

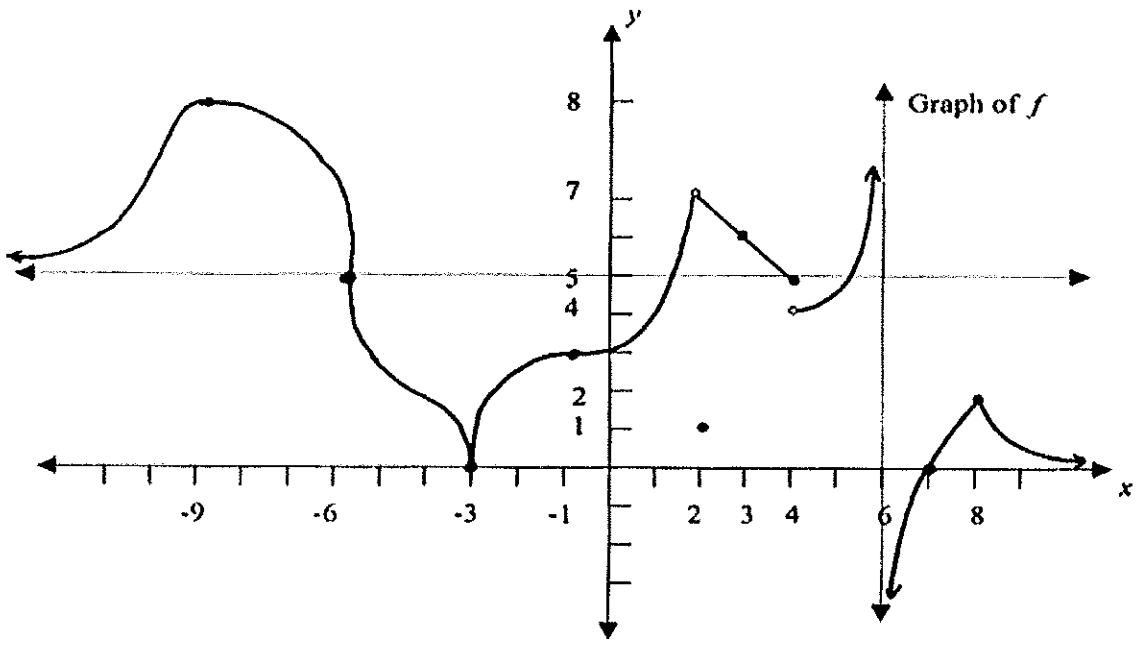
99. $f(x) = \begin{cases} x + 5\,;\ x \leq 2 \\ -4\,;\ x > 2 \end{cases}$



100. $f(x) = \begin{cases} x - 1; x \leq -2 \\ 2x - 1;\ -2 < x \leq 4 \\ -3x + 8; x > 4 \end{cases}$



Use the graph below to answer questions 101-116



101. $\lim_{x\to -3^-} f(x) =$

102. $\lim_{x\to 4^+} f(x) =$

103. $\lim_{x\to -3^+} f(x) =$

104. $\lim_{x\to 4} f(x) =$

105. $\lim_{x\to -3} f(x) =$

106. $\lim_{x\to 6^-} f(x) =$

107. $\lim_{x\to -6^+} f(x) =$

108. $\lim_{x\to 2^-} f(x) =$

109. $\lim_{x\to -6} f(x) =$

110. $\lim_{x\to 2^+} f(x) =$

111. $\lim_{x\to 2} f(x) =$

112. $\lim_{x\to -\infty} f(x) =$

113. $\lim_{x\to 4^-} f(x) =$

114. $\lim_{x\to \infty} f(x) =$

115. $f(2)$

116. $f(3) =$